Color Models:

HSV, HSB, HSL, RGB, CMYK

Image Files:

PNG, JPG, GIF, SVG, EPS, AI

Intensity Transforms:

$S = L - 1 - r$: invert

$S = c\log(1+r)$: Low or High

$S = c\log r^{-1}s$: inverse

$S = cr^{\gamma}$, $\gamma > 1$ darkness, $\gamma < 1$ brightness

r = input pixel intensity

S = output   L = Max intensity

Bit Plane: Low # planes high # for small feats large-scale + noisy

Histogram:

$L = 2^K$

1: Find $n_K = n(r_K)$

2: PDF and CDF   x Bit img

3: $(L-1)$CDF

4: Round to Determine where it goes

Image Derivitives:

$\partial f / \partial x = f(x+1, y) - f(x, y)$

$\partial f / \partial y = f(x, y+1) - f(x, y)$

Robert: $g_x = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ $g_y = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Prewitt: $g_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ $g_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Sobel: $g_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $g_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

2D Projective Space:

point at infinity : $x_{ideal} = [x, y, 0]^T$

line at inf : $l_\infty [0, 0, 1]^T$

x is on the line iff $l^T x = x^T l = 0$

intersection of lines at $x = l \times l'$

$l \times l' = \begin{vmatrix} i & j & K \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{pmatrix} b_1c_2 - b_2c_1 \\ a_1c_2 - a_2c_1 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

3D projective space:

$X_{ideal} = [x_1, x_2, x_3, 0]^T$

- Plane at infinity

$\pi_\infty = [0, 0, 0, 1]^T$

$\pi^T x = \pi x^T = 0$ 3 π_i define point

$3x_i$ define plane

Quadric $Q \to x^T Q x = 0$

Dual quadric $Q^* \to \pi^T Q^* \pi = 0$

$X' = HX$, $\pi' = H^{-T}\pi$

$Q' = H^{-T} Q H^{-1}$

$Q^{*'} = H Q^* H^T$

Matrix Properties:

$(AB)^T = B^T A^T$

$(A^T)^T = A$

$(A^{-1})A = I$ ← indent Matrix

Hierarchy of Transformation:

Class 1: Isometrics, H_E include euclidean Rotation and Transformation preserves orientation

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} \epsilon\cos\theta & -\sin\theta & t_x \\ \epsilon\sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \quad \epsilon = 1, \ \epsilon = -1$$

Class 2: Similarity, 1 + uniform Scaling

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

4 DoF: rot, scale, 2 translate

Class 3: Affine, 1 + nonuni-scale + shear

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

6 DoF: 2 rot, 2 Scale, 2 translation

Class 4: Projective   1 + projective

8 DoF

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$$

$$H = H_S H_A H_P = \begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} A & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} I & 0 \\ v^T & v \end{bmatrix}$$

Recovery of Affine and Metric Props 2D

Under Projective, ideal points → Finite points

$x' = H_P x_{ideal} = \begin{bmatrix} A & t \\ v^T & v \end{bmatrix} [x_1, x_2, 0]$
$= [A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, v_1x_1 + v_2x_2]$

Under Affine trans: l_∞ remains at ∞

$l' = H_A^{-T} l_\infty = \begin{bmatrix} A^{-T} & 0 \\ -t^TA^{-T} & 1 \end{bmatrix} [0, 0, 1]^T = [0, 0, 1]^T$

Ideal point maps to ideal point but moves

$H_A x_{ideal} = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix} [x_1, x_2, 0] = [A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, 0]^T$

under circular points $I = (1, i, 0)$ and $J = (1, -i, 0)$ on l_∞ are fixed

$I' = H_S I = \begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix} [1, i, 0] = se^{-i\theta}[1, i, 0]$

- dual to circular points is $C^*_\infty = IJ^T + JI^T$

- given two lines, l and m, angle is

$$\cos\theta = \frac{l^T C^*_\infty m}{\sqrt{(l^T C^*_\infty l)(m^T C^*_\infty m)}}$$

Conic and Dual Conic

Conic rep'd in R^2 : $ax^2 + bxy + cy^2$

Point conic C rep'd by $x^T C x = 0$

line $l = Cx$ tangent to C at x on C

a line dual conic $C^* \to l^T C^* l = 0$

$$C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & F \end{bmatrix}$$

2D Projective Transformation

point transform defined as $x' = Hx$

↳ line l transform to $l' = H^{-T} l$

$l'^T x_i' = 0 \to l^T H^{-1} H x_i = 0 \to l^T (H^{-1}H) x_i = 0$

↳ Conic C transforms to: $C' = H^{-T} C H^{-1}$

$x' = Hx \to H^{-1}x' = x$

$x^T C x = x'^T H^{-T} C H^{-1} x' \to x^T C x = x'^T (H^{-T} C H^{-1})$

↳ Dual Conic C^* transforms to: $C^{*'} = H C^* H^T$

Recovery in 3D

under H_A, π_∞ remains at $\pi' = H_A^{-T}$ $\pi_\infty = [0, 0, 0, 1]^T$

under H_S, absolute conic Ω_∞ on π_∞ are fixed

Absolute conic corresponds to conic w/ identity matrix

$C = I$

Given two lines, θ on a projective plane

$$\cos\theta = \frac{(d_1^T \Omega_\infty d_2)}{\sqrt{(d_1^T \Omega_\infty d_1)(d_2^T \Omega_\infty d_2)}}$$

Finite Cameras

$R^3 \to R^2$ is given by $x = PX$

homogeneous projective P

where $t = -RC$

$P = K[R | t]$ $P = KR[I | -\tilde{C}]$

Camera intrinsic matrix K

$$\begin{bmatrix} a_x & s & x_0 \\ & a_y & y_0 \\ & & 1 \end{bmatrix}$$

a_x, a_y are focal length, $a_x = a_y$ for square

$x_0 = m_x p_x$, $y_0 = m_y p_y$ } principal point

Camera Projection Matrix P

Camera Center is Right Null Space since $PC = 0$

$P = [M | P_4] = [P_1 P_2 P_3 P_4] = \begin{bmatrix} P_1^T \\ P_2^T \\ P_3^T \end{bmatrix}$

First 3 columns : $P_1 P_2 P_3$ rep' vanishing points in x, y, z

$x = PD = [P_1 P_2 P_3 P_4][x, y, z, D]$

First 2 Rows P_1^T P_2^T rep planes defined by census and lines $x = 0$, $y = 0$

P_3^T is principal plane

$-C = (x, y, z, T)^T$

$X = det(P_{234})$, $Y = det(P_{134})$

$Z = det(P_{124})$, $T = det(P_{123})$

K and R can be obtained by (→ 3x3 Matrix, → cam rot)

Image Definition → $M = KR$ via RQ decomposition, $M = R'Q$ → 4x4 Symmetric Matrix

$K = R'$, $R = Q$

Translation vector t is found by $P_4 = Kt$ or $t = K^{-1}P_4$

Fwd projection converts world point to image point $P_X = X$

↳ Let x rep $D = (d^T, 0)^T$

Two representing vanishing points only affected by first 3 columns of P $x = PD = [M | P_4] D = Md$

Backward projection $P_2 \to P_3$, $P_X = x$

↳ line back projects to a plane

where $\pi = P^T l$, $x^T l = 0 \to (PX)^T l = 0$

$x^T P^T l = 0 \to x^T \pi = 0$

↳ Cone back projects to quadratic

where $Q_{Co} = P^T C P$

$x^T C x = 0 \to (PX)^T C (PX) = 0$

$x^T P^T C P x = 0 \to x^T Q_{\infty} x = 0$

Robert, Prewitt and Sobel pros, cons.

R - Pros → Very fast, detects sharp edges
  cons → Sensitive to noise

P - pro → more Robust, easy to compute
  Cons → Not Noise resistant,

S - pro → more widly used, more stable
  cons → computationally expensive,

Manual Second Derivitive:

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) + 4F(x, y)$$

Homography Relations

Images of coplanar 3D points : $H = K[r_1, r_2, t]$

↳ ex $X = (x, y, 0, 1)^T$ is a set of coplanar points from XY0 coordinate plane

↳ Let $P = K[R | t]$ be $P = K[r_1, r_2, r_3, t]$

↳ $x = PX = x = K[r_1, r_2, t]X$ ∴ $Z = 0$

Images obtained w/ same camera center may be mapped to one another by a homography : H

Two images w/ same center : $H = K'R'(KR)^{-1}$

1. Consider $P = K[I | 0]$, $P' = K'[I | 0]$
2. Isolate $\to K^{-1}P = [I / 0]$
3. Sub $P' \to P' = K'K^{-1}P$
4. Let $x' = P'x$ and image points

$x' = K'K^{-1}x$

Camera Calibration

Let x represent $X_\infty = (d^T, 0)^T$

↳ mapping $x = PX_\infty$ is given by homography

Given $C' = H^{-T} C H^{-1}$, abs conic transforms to $\Omega'_\infty = H^{-T}\Omega_\infty H^{-1}$

↳ Since ∞ corresponds to $C = I$ IAC: $w = H^{-T} H^{-1} I$

Practice Problems:

Given a 1D image:

$f'(x) = f(x+1) - f(x)$
$f''(x) = f'(x) - f(x) + f(x-1)$

$f''(3) = f(4) - 2(f(3)) + f(2)$
$= 5 - 2(6) + 6$
∴ $f''(3) = -1$

$f'(3) = f(4) - f(3)$
$= 5 - 6$ ∴ $f'(3) = -1$

Add padding before and after

$x = 0$: $f(x) = 6$, $f'(x) = 0$, $f''(x) = -6$

x	f(x)	f'(x)	f''(x)
1	6	0	0
2	6	0	9
3	6	-1	-1
4	5	-1	9
5	4	-1	0
6	3	-1	0
7	2	-1	0
8	1	0	1
9	1	0	0
10	1	0	0
11	1	0	0
12	1	0	0
13	1	5	5
14	6	0	-5
15	6	0	0
16	6	0	0
17	6	0	0
18	6	/	/

0	1	0
1	-4	1
0	1	0

0	2	5	7
2	5	7	3
5	6	3	1
5	2	1	0

→ ADD Padding 0's on outside

1) Writedown equations of first order Derivitives:

$$\frac{\delta f}{\delta x} = f(x+1, y) - f(x, y)$$

$$\frac{\delta f}{\delta y} = f(x, 1+y) - f(x, y)$$

2) Compute first Derivitive:

$\frac{\delta f}{\delta x}$ is for Rows where y is the same

Row $y = 0$: $[0-0, 2-0, 5-2, 7-5, 0-7]$

$\frac{\delta f}{\delta y}$ is for Cols where x is the same

Col $x = 0$: $[0-0, 2-0, 5-2, 5-5, 0-5]$

3) Calculate laplacian of 3Bit image

$$\nabla^2 f = \frac{\delta^2 f}{\delta^2 x^2} + \frac{\delta^2 f}{\delta y^2}$$

Translate the 010 / 1-41 / 010 Matrix across padded Matrix

$\boxed{111 = 7}$ 3bits

Normalize function $N'_{ij} = round\left(7 \times \frac{V_{ij} - min(V)}{max(V) - min(V)}\right)$

$min(V) = -20$ $max(v) = 4$

Translate again

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3 bit Histogram, Compute $P_r(r_k)$

12473
24731
56211
47111

r_k	n_k	PDF	Dec	CDF(s_k)	$(L-1)s_k$	Round
0	0	0/20	0	0	0	0
1	7	7/20	0.35	0.35	2.45	2
2	3	3/20	0.15	0.5	3.5	4
3	2	2/20	0.1	0.6	4.2	4
4	3	3/20	0.15	0.75	5.25	5
5	1	1/20	0.05	0.8	5.6	6
6	1	1/20	0.05	0.85	5.95	6
7	3	3/20	0.15	1	7	7

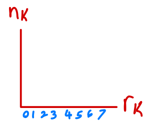
use Rounded values to recreate

24574
45742
66422
57222

Equalized Histogram

r_k	n_k
0	0
1	0
2	7
3	0
4	5
5	3
6	2
7	3

Graph Red boxes



General Form of camera proj. matrix

① $P = K[R|T]$: $K = \begin{bmatrix} a_x & s & x_0 \\ & a_y & y_0 \\ & & 1 \end{bmatrix}$, $R|T$: $\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

② Geometrically the last row (n^T, d) represents the plane at infinity aka principle plane, $n^T X_{xyz} + d = P$
P is ⊥ to camera's viewing direction.

③ 10 DoF, general cam has 11 DoF, 5 intrinsic (a_x, a_y, s, x_0, y_0). Geometric (rst) $s = 0 \to 6+4$

2D projective space, Affine transformation

① prove Affine transformation maps an ideal point
② Convert eq of circle: $(x-2)^2 + (y-3)^2 = 9$ to homo
③ Verify Dual Conic denoted by $C^* = C^{-1}$

① $x' = H_A x_{ideal}$
$= \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix} [x_1, x_2, 0]^T$
$= \left[A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, 0\right]^T$

$l' = H_A^{-T} l_\infty$
$= \begin{bmatrix} A & 0 \\ -t^T & A^{-T} \; 1 \end{bmatrix} [0,0,1]^T$
$= [0,0,1]^T$

② $(x-2)^2 + (y-3)^2 = 9$
$= x^2 + y^2 - 4x - 6y + 4 = 0$
$x \to x/z \quad y \to y/z$
$x^2 + y^2 - 4xz - 6yz + 4z^2 = 0$
$C = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ -2 & -3 & -4 \end{pmatrix}$

④ plus $z = 0$
$\therefore x^2 + y^2 = 0$
$\to [i, \pm j, 0]$
$= I$ or J

③ $C^* = adj(C)$
$= det(C)C^{-1}$

Fit 3 points into a line (1,1)(2,3)(3,5) using least squares

n = # of points = 3

$$m = \frac{n\sum(x \cdot y) - (\sum x)(\sum y)}{n\sum(x^2) - (\sum x)^2} = \frac{3(1+6+15) - (1+2+3)(1+3+5)}{3(1+4+9) - (1+2+3)^2}$$

$$= \frac{66 - 6(9)}{42 - 36} = 2$$

$$b = \frac{\sum y - m \sum x}{n}$$

$$b = \frac{(1+3+5) - 2(1+2+3)}{3} = \frac{9 - 12}{3} = -1$$

$\therefore y = mx + b$
$y = 2x - 1$

1D image filter using convolution, Pad with 0's

f: 32142   w = 213
↳ flip = 312

0003214200 0
312 = 0
312 = 6
312 = 7
312 = 13
312 = 15
312 = 11
312 = 14
312 = 6
312 = 0

$\therefore f * w$ [0, 6, 7, 13, 15, 11, 14, 6, 0]

Krish Patel